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ON STRATEGIES IN DIFFERENTIAL GAMES*

G. V. TOMSKII

Piecewise-programmed, piecewise-synthesizing and recursive strategies in differential games are examined. It is shown that in a specific sense these strategies can be considered as special cases of upper Δ-strategies. The paper borders on the studies in /1—8/.

1. Let the dynamics of a game be described by the vector differential equation

$$dx/dt = f(t, x, u, v), \quad t_0 \leqslant t \leqslant T, \quad x \in R^n, \quad u \in P(t) \subset U, \quad v \in Q(t) \subset V \tag{1.1}$$

where $U(V)$ is a compact set in Euclidean space R^p (R^q) and at least one pair of controls $u(t)$ and $v(t)$ measurable on $[t_0, T]$ exists, such that $u(t) \in P(t)$, $v(t) \in Q(t)$, $t_0 \leqslant t \leqslant T$. The function f on the right-hand side of the motion Eqs. (1.1) is continuous on $[t_0, T] \times R^n \times U \times V$ and on this set satisfies a Lipschitz condition in x with a constant λ. We shall examine two controlled dynamic systems /6/ governed by Eq. (1.1).

Dynamic system $\Sigma_1 = ([t_0, T], R^n, D_1, D_2, \varkappa)$. The set D_1 (D_2) of admissible controls of the first (second) player in system Σ_1 consists of all vector-valued functions $u(t)(v(t))$ measurable on interval $[t_0, T]$, satisfying the conditions $u(t) \in P(t)$ $(v(t) \in Q(t))$, $t_0 \leqslant t \leqslant T$. The paths $x(t) = \varkappa(t, t_*, x_*, u, v)$ of this system are defined as the solutions of the system of Eqs. (1.1) when $u = u(t) \in D_1$ and $v = v(t) \in D_2$ under the initial condition $x(t_*) = x_*$.

Dynamic system $\Sigma_2 = ([t_0, T], R^n, D_1(k_1), D_2(k_2), \varkappa)$. The set $D_1(k_1)$ $(D_2(k_2))$ of admissible controls of the first (second) player consists of all vector-valued functions $u(t, x)(v(t, x))$ defined on $[t_0, T] \times R^n$, taking values in $U(V)$, $u(t, x) \in P(t) \subset U$ $(v(t, x) \in Q(t) \subset V)$, $t_0 \leqslant t \leqslant T$, $x \in R^n$, measurable in t on $[t_0, T]$ for each fixed x, and satisfying a Lipschitz condition in x with constant $k_1(k_2)$ on set $[t_0, T] \times R^n$. The set $D_1(k_1)$ $(D_2(k_2))$ can be looked upon as a set consisting of mappings of interval $[t_0, T]$ into the set of functions

$$U_1 = \{u(x) \in C[R^n, \; U] \mid \|u(x_1) - u(x_2)\| \leqslant k_1 \|x_1 - x_2\|, \quad \text{for all} \quad x_1, x_2 \in R^n\}$$
$$(V_1 = \{v(x) \in C[R^n, \; V] \mid \|v(x_1) - v(x_2)\| \leqslant k_2 \|x_1 - x_2\|, \quad \text{for all} \quad x_1, x_2 \in R^n\})$$

The paths $x(t) = \varkappa(t, t_*, x_*, u, v)$ of system Σ_2 are defined as the solutions of the system of Eqs.(1.1) when $u = u(t, x) \in D_1(k_1)$ and $v = v(t, x) \in D_2(k_2)$ under the initial condition $x(t_*) = x_*$. It is assumed that function f on the right hand side of the motion Eqs. (1.1) satisfy on set $[t_0, T] \times R^n \times U \times V$ a Lipschitz condition in x, u, v with a constant λ.

2. Piecewise-programmed strategies /6,7/ in system Σ_2 will be called piecewise-synthesizing strategies. By $D_1^*[k_1, t_*]$ $(D_2^*[k_2, t_*])$ we denote the set of all piecewise-synthesizing strategies of the first (second) player in the quasidynamic system $\Sigma_2(t_*, x_*)$ /6/. Let $\Delta = \{t_* = t_0^\Delta < t_1^\Delta < \ldots < t_{n(\Delta)}^\Delta = T\}$ be any finite partitioning of interval $[t_*, T]$. By $D_1^\Delta[t_*]$ $(D_2^\Delta[t_*])$ we denote the set of all upper Δ-strategies, by $D_{1\Delta}[t_*]$ $(D_{2\Delta}[t_*])$ we denote the set of all Δ-strategies, and by $D_1^*[t_*]$ $(D_2^*[t_*])$ we denote the set of all piecewise-programmed strategies of the first (second) player in system $\Sigma_1(t_*, x_*)$ /6/. The following statement is valid.

Theorem 1. For any piecewise-synthesizing strategy $\varphi \in D_1^*[k_1, t_*]$ $(\psi \in D_2^*[k_2, t_*])$ there exists an upper Δ-strategy $\varphi^\Delta \in D_1^\Delta[t_*]$ $(\psi^\Delta \in D_2^\Delta[t_*])$ such that

$$\varkappa(t, t_*, x_*, \varphi, \psi_\Delta) = \varkappa(t, t_*, x_*, \varphi^\Delta, \psi_\Delta)$$

for all Δ-strategies $\psi_\Delta \in D_{2\Delta}[t_*]$

$$(\varkappa(t, t_*, x_*, \varphi_\Delta, \psi) = \varkappa(t, t_*, x_*, \varphi_\Delta, \psi^\Delta)$$

for all Δ-strategies $\varphi_\Delta \in D_{1\Delta}[t_*]$).

3. Let

$$S(t_*) = \bigcup_{t_*<t<T} [D_1[t_*, t) \times D_2[t_*, t)], \quad \Pi(t_*) = \bigcup_{\substack{t_*<t<T \\ t<\theta\leqslant T}} D_1[t, \theta)$$

Definition 1. Any finite collection of mappings $a = (a_1, \ldots, a_n)$, where $a = a_1 \in D_1[t_*, T)$ for $n = 1$ and

$$a_1 \in \bigcup_{t_*<t<T} D_1[t_*, t), \; a_k : S(t_*) \to \Pi(t_*), \; k = 2, \ldots, n$$

for $n \geqslant 2$, and where the conditions $a_n(u_t, v_t) \in D_1[t, T)$ and $a_k(u_t, v_t) \in D_1[t, \theta)$, $t < \theta \leqslant T$, $k = 1, 2, \ldots, n-1$, are fulfilled if $\{u_t, v_t\} \in D_1[t_*, t) \times D_2[t_*, t)$, $t_* < t < T$, is called the first player's recursive strategy in system $\Sigma_1(t_*, x_*)$. The second player's recursive strategy $b = (b_1, \ldots, b_m)$ in system $\Sigma_1(t_*, x_*)$ is defined analogously.

The path $x(t) = \varkappa(t, t_*, x_*, a, b)$ of system Σ_1, generated by a pair of recursive strategies

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$a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_m)$, is determined as follows. At the initial instant t_* the players choose the controls

$$u_1 = a_1 \in D_1 [t_*, t_{11}), \quad v_1 = b_1 \in D_2 [t_*, t_{21})$$

For definiteness let $t_{11} < t_{21}$. Then at instant t_{11} the first player chooses the control $u_2 = a_2 (u_1, v_{11}) \in D_1 [t_{11}, t_{12})$, depending on the controls u_1 and v_{11} realized by the players on interval $[t_*, t_{11}]$, where v_{11} denotes the restriction of control $v_1 = b_1$ on the interval $[t_*, t_{11})$. (We note that in contrast to peicewise-programmed strategies the instant t_{12}, in general, also depends on controls u_1 and v_{11}: $t_{12} = t_{12} (u_1, v_{11})$). We compare the quantities t_{21} and t_{12}. If $t_{21} < t_{12}$, then at instant t_{21} the second player chooses the control

$$v_2 = b_2 (u_1, u_{21}, v_1) \in D_2 [t_{21}, t_{22} (u_1, u_{21}, v_1))$$

depending on the controls (u_1, u_{21}) and v_1 realized by the players on the interval $[t_*, t_{21})$ (u_{21} denotes the restriction of control u_2 on the interval $[t_{11}, t_{21})$). If $t_{21} > t_{12}$, then at instant t_{12} the first player chooses the control

$$u_3 = a_3 (u_1, u_2, v_{12}) \in D_1 [t_{12}, t_{13} (u_1, u_2, v_{12}))$$

where v_{12} denotes the restriction of control v_1 on $[t_*, t_{12})$.

Continuing this process, in at most $n + m - 1$ steps we obtain uniquely the pair of controls

$$u = (u_1, \ldots, u_n) = u (a, b), \quad v = (v_1, \ldots, v_m) = v (a, b)$$

generated by the pair of strategies a and b. Thus, a pair of recursive strategies a and b determines a unique path $x (t) = \varkappa (t, t_*, x_*, a, b) = \varkappa (t, t_*, x_*, u (a, b), v (a, b))$ of system Σ_1. The choice of recursive stategy $a = (a_1, \ldots, a_n)$ by the first player signifies that in the course of the game he can change his control n times, depending on the information at hand. The control switching instants are not fixed at the start of the game as when applying piecewise-programmed strategies, but are determined by the player during the game.

Notes. 1°. Any finite collection of mappings $a = (a_1, \ldots, a_n)$, where

$$a = a_1 \in \mathcal{D}_1, \ n = 1, \quad a_1 \in \bigcup_{t_* < t < T} D_1 [t_*, t), \ a_k: [t_*, T] \times R^n \to \Pi (t_*), \quad n \geqslant 2, \ k = 2, \ldots, n$$

satisfying the following conditions:

$$a_n (t, x) \in D_1 [t, T), \ a_k (t, x) \in D_1 [t, \theta), \ t < \theta < T, \ k = 1, \ldots, n - 1$$

is called the first player's positional recursive strategy in system $\Sigma_1 (t_*, x_*)$ (see /8/). The second player's positional recursive strategy is defined in the same manner. Any positional recursive strategy $a = (a_1, \ldots, a_n)$ induces a recursive strategy $a = (a_1, a_2^*, \ldots, a_n^*)$, where

$$a_k^* (u_t, v_t) = a_k (t, \varkappa (t, t_*, x_*, u_t, v_t), \ k = 2, \ldots, n$$

thus, positional recursive strategies are special cases of recursive strategies.

2°. A pair $\varphi = (\Delta, \varphi_\Delta)$, where Δ is any finite partitioning of interval $[t_*, T]$ and φ_Δ is a first player's recursive strategy $\varphi_\Delta = (\varphi_{\Delta,1}, \ldots, \varphi_{\Delta,n(\Delta)})$ such that $\varphi_{\Delta,k} (u_{k-1}, v_{k-1}) \in D_1 [t_{k-1}^\Delta, t_k^\Delta)$ if

$$(u_{k-1}, v_{k-1}) \in D_1 [t_*, t_{k-1}^\Delta) \times D_2 [t_*, t_{k-1}^\Delta), \ k = 2, \ldots, n (\Delta)$$

is called the first player's piecewise-programmed strategy in system $\Sigma_1 (t_*, x_*)$ (see /3,6,7/). In analogous fashion we can rephrase the definition of piecewise-programmed strategies for the second player. Consequently,

$$D_k^* [t_*] \subset D_k^r [t_*], \quad k = 1,2$$

where $D_k^r [t_*]$ is the set of all recursive strategies of the k-th player in system $\Sigma_1 (t_*, x_*)$.

We obtain the next statement by comparing the definitions of recursive and upper Δ-strategies.

Theorem 2. For any finite partitioning Δ of interval $[t_*, T]$ and any recursive strategy $a \in D_1^r [t_*]$ ($b \in D_2^r [t_*]$) an upper Δ-strategy $\varphi^\Delta \in D_1^\Delta [t_*]$ ($\psi^\Delta \in D_2^\Delta [t_*]$) exists such that

$$u (a, \psi_\Delta) = u (\varphi^\Delta, \psi_\Delta), \quad v (a, \psi_\Delta) = v (\varphi^\Delta, \psi_\Delta), \quad (u (\varphi_\Delta, b) = u (\varphi_\Delta, \psi^\Delta), \quad v (\varphi_\Delta, b) = v (\varphi_\Delta, \psi^\Delta)) \tag{3.1}$$

for all $\psi_\Delta \in D_{2\Delta} [t_*]$ ($\varphi_\Delta \in D_{1\Delta} [t_*]$).

Proof. We take an arbitrary recursive strategy $a = (a_1, \ldots, a_n) \in D_1^r [t_*]$ and any finite partitioning Δ of interval $[t_*, T]$. We need to show that with the use of strategy a we can construct an upper Δ-strategy $\varphi^\Delta = (\varphi^{\Delta,1}, \ldots, \varphi^{\Delta, n(\Delta)})$ in system $\Sigma_1 (t_*, x_*)$, satisfying relations (3.1). We indicate a method for constructing the mapping

$$\varphi^{\Delta,1}: D_2 [t_*, t_1^\Delta) \to D_1 [t_*, t_1^\Delta) \tag{3.2}$$

Let $a_1 \in D_1 [t_*, t_1)$. If $t_1 \geqslant t_1^\Delta$, then $\varphi^{\Delta,1}$ is the restriction of control a_1 on interval $[t_*, t_1^\Delta)$. In this case $\varphi^{\Delta,1}$ is independent of the control chosen by the second player on $[t_*, t_1^\Delta]$. Let $t_1 < t_1^\Delta$ and $a_2 (a_1, v_1) \in D_1 [t_1, t_2)$. If $t_2 = t_2 (a_1, v_1) \geqslant t_1^\Delta$, then $\varphi^{\Delta,1} = a_1$ on interval $[t_*, t_1)$, while on interval $[t_1, t_1^\Delta)$ it coincides with the restriction of control $a_2 (a_1, v_1)$ on this interval. If on interval $[t_*, t_1)$ the second player chose a control v_1 such that $t_2 = t_2(a_1, v_1) < t_1^\Delta$, but the condition

$$t_3 = t_3(a_1,\ a_2(a_1,\ v_1);\ v_2) \geqslant t_1^\Delta$$

is valid for the second player's control on interval $[t_*, t_2)$, then $\varphi^{\Delta,1} = a_1$ on interval $[t_*, t_1]$, $\varphi^{\Delta,1} = a_2(a_1, v_1)$ on interval $[t_1, t_2)$, and on $[t_2, t_1^\Delta)$ the mapping $\varphi^{\Delta,1}$ coincides with the restriction of mapping $a_3(a_1, a_2(a_1, v_1); v_2)$. Continuing these arguments, we construct the mapping (3.2). In analogous manner we can construct the mappings $\varphi^{\Delta,k}$, $k = 2, \ldots, n(\Delta)$. The theorem is proved.

4. Let us consider recursive strategies in system $\Sigma_2(t_*, x_*)$. Let

$$S_1(t_*) = \bigcup_{t_*<t<T} [D_1[k_1, t_*, t) \times D_2(k_2, t_*, t)], \quad \Pi_1(t_*) = \bigcup_{\substack{t_*<t<T \\ t<\theta\leqslant T}} D_1[k_1, t, \theta)$$

Definition 2. Any finite collection of mappings $a = (a_1, \ldots, a_n)$, where

$$a = a_1 \in D_1[k_1,\ t_*,\ T), \quad n = 1, \quad a_1 \in \bigcup_{t_*<t<T} D_1[k_1, t_*, t),\ a_k: S_1(t_*) \to \Pi_1(t_*)\ n \geqslant 2,\ k = 2, \ldots, n$$

and conditions

$$a_n(u_t,\ v_t) \in D_1[k_1,\ t,\ T), \quad a_k(u_t,\ v_t) \in D_1[k_1,\ t,\ \theta), \quad t < \theta \leqslant T, \quad k = 1, 2, \ldots, n-1$$

are fulfilled if $\{u_t, v_t\} \in D_1[k_1, t_*, t) \times D_2[k_2, t_*, t)$, $t_* < t < T$, is called the first player's recursive strategy in system $\Sigma_2(t_*, x_*)$.

The second player's recursive strategies in system $\Sigma_2(t_*, x_*)$ are defined analogously. By $D_1^r[k_1, t_*]$ ($D_2^r[k_2, t_*]$) we denote the set of all recursive strategies of the first (second) player in system $\Sigma_2(t_*, x_*)$. The inclusions

$$D_i^*[k_i,\ t_*] \subset D_i^r[k_i,\ t_*], \quad i = 1, 2$$

are valid. The following statement can be obtained by combining the methods of proofs of Theorems 1 and 2.

Theorem 3. For any finite partitioning Δ of interval $[t_*, T]$ and any recursive strategy $a \in D_1^r[k_1, t_*]$ ($b \in D_2^r[k_2, t_*]$) an upper Δ-strategy $\varphi^\Delta \in D_1^\Delta[t_*]$ ($\psi^\Delta \in D_2^\Delta(t_*)$) exists such that

$$\varkappa(t,\ t_*,\ x_*,\ a,\ \psi_\Delta) = \varkappa(t,\ t_*,\ x_*,\ \varphi^\Delta,\ \psi_\Delta)$$

$$(\varkappa(t,\ t_*,\ x_*,\ \varphi_\Delta,\ b) = \varkappa(t,\ t_*,\ x_*,\ \varphi_\Delta,\ \psi^\Delta))$$

for all $\psi_\Delta \in D_{2\Delta}[t_*]$ ($\varphi_\Delta \in D_{1\Delta}[t_*]$).

Analogous statements are valid for global strategies /9/.

5. It can be proved that the sets $\Phi(\Sigma_1, t_*, x_*)$ and $\Phi(\Sigma_2, t_*, x_*)$ of all paths of systems $\Sigma_1(t_*, x_*)$ and $\Sigma_2(t_*, x_*)$ coincide, i.e.

$$\Phi(t_*,\ x_*) = \Phi(\Sigma_1,\ t_*,\ x_*) = \Phi(\Sigma_2,\ t_*,\ x_*)$$

(if function f satisfies a Lipschitz condition in (x, u, v)). Let a certain functional (the second player's gain) H be specified on set $\Phi(t_*, x_*)$. Then we have defined the second player's gain function

$$K(\varphi^\Delta,\ \psi_\Delta) = H(\varkappa(\cdot,\ t_*,\ x_*,\ \varphi^\Delta,\ \psi_\Delta)) \quad \text{on set} \quad D_1^\Delta[t_*] \times D_{2\Delta}[t_*]$$

$$K(\varphi_\Delta,\ \psi^\Delta) = H(\varkappa(\cdot,\ t_*,\ x_*,\ \varphi_\Delta,\ \psi^\Delta)) \quad \text{on set} \quad D_{1\Delta}[t_*] \times D_2^\Delta[t_*]$$

$$K(\varphi,\ \psi) = H(\varkappa\ \cdot,\ t_*,\ x_*,\ \varphi,\ \psi)) \quad \text{on set} \quad D_1^*[k_1,\ t_*] \times D_2^*[k_2,\ t_*]\ (D_i^*[t_*] \subset D_i^*[k_i,\ t_*],\ i = 1, 2),$$

$$K(a,\ b) = H(\varkappa(\cdot,\ t_*,\ x_*,\ a,\ b)) \quad \text{on set} \quad D_1^r[k_1,\ t_*] \times D_2^r[k_2,\ t_*] \subset (D_i^r[t_*]\, D_i^r[k_i,\ t_*],\ i = 1,\ 2).$$

Let us consider the antagonistic differential games:

$$\Gamma_1(t_*,\ x_*) = \langle D_1^*[t_*],\ D_2^*[t_*],\ K\rangle$$

in the class of piecewise-programmed strategies,

$$\Gamma_2(t_*,\ x_*) = \langle D_1^*[k_1,\ t_*],\ D_2^*[k_2,\ t_*],\ K\rangle$$

in the class of piecewise-synthesizing strategies,

$$\Gamma_3(t_*,\ x_*) = \langle D_1^r[t_*],\ D_2^r[t_*],\ K\rangle, \quad \Gamma_4(t_*,\ x_*) = \langle D_1^r[k_1,\ t_*],\ D_2^r[k_2,\ t_*],\ K\rangle$$

in the class of recursive strategies. By Theorem 1 we have

$$V^\Delta(t_*, x_*) = \inf_{\varphi_\Delta \in D_{1\Delta}[t_*]} \sup_{\psi^\Delta \in D_2^\Delta[t_*]} K(\varphi_\Delta, \psi^\Delta) \geqslant \inf_{\varphi_\Delta \in D_{1\Delta}[t_*]} \sup_{\psi \in D_2^*[k_2, t_*]} K(\varphi^\Delta, \psi) \geqslant$$

$$\inf_{\varphi \in D_1^*[k_1, t_*]} \sup_{\psi \in D_2^*[k_2, t_*]} K(\varphi, \psi) \geqslant \sup_{\psi \in D_2^*[k_2, t_*]} \inf_{\varphi \in D_1^*[k_1, t_*]} K(\varphi, \psi) \geqslant \sup_{\psi_\Delta \in D_{2\Delta}[t_*]} \inf_{\varphi^\Delta \in D_1^\Delta[t_*]} K(\varphi^\Delta, \psi_\Delta) = V_\Delta(t_*, x_*)$$

The inequalities

$$V^\Delta(t_*, x_*) \geqslant \inf_{a \in D_1^r[t_*]} \sup_{b \in D_2^r[t_*]} K(a, b) \geqslant \sup_{a \in D_2^r[t_*]} \inf_{a \in D_1^r[t_*]} K(a, b) \geqslant V_\Delta(t_*, x_*)$$

$$V^{\Delta}(t_*, x_*) \geqslant \inf_{a\in D_1^r[k_1, t_*]} \sup_{b\in D_1^r[k_2, t_*]} K(a, b) \geqslant \sup_{b\in D_2^r[k_2, t_*]} \inf_{a\in D_1^r[k_1, t_*]} K(a, b) \geqslant V_{\Delta}(t_*, x_*)$$

follow from Theorems 2 and 3. Thus, if

$$\inf_{\Delta} V^{\Delta}(t_*, x_*) = \sup_{\Delta} V_{\Delta}(t_*, x_*) \tag{5.1}$$

then all games $\Gamma_k(t_*, x_*)$, $k = 1,2,3,4$, have the value

$$\text{val}\,\Gamma_1(t_*, x_*) = \text{val}\,\Gamma_k(t_*, x_*), \quad k = 2,3,4$$

It is well known that if H is a uniformly continuous functional on set $\Phi(t_*, x_*)$ (see /10/), then for the fulfillment of condition (5.1) it suffices to require that the function f on the right hand side of motion Eqs. (1.1) satisfy the condition

$$\inf_{v\in D_2} \sup_{u\in D_1} \int_{t_1}^{t_2} \langle l, f(t_1, x, u(s), v(s))\rangle ds - \sup_{u\in D_1} \inf_{v\in D_2} \int_{t_1}^{t_2} \langle l, f(t_1, x, u(s), v(s))\rangle ds \leqslant \gamma(t_2 - t_1), \quad \lim_{\delta\to 0} \frac{\gamma(\delta)}{\delta} = 0 \tag{5.2}$$

for all l, $x \in R^n$, $t_0 \leqslant t_1 < t_2 < T$. This condition is fulfilled, for example, if

$$f(t, x, u, v) = f_1(t, x, u) + f_2(t, x, v)$$

where f_1 and f_2 are continuous vector-valued functions. If $P(t) = U$ and $Q(t) = V$ for all $t_* \leqslant t \leqslant T$, then it follows from the saddle point condition for a small game /1/.

6. Let certain sets M and N exist in $[t_0, T] \times R^n$ and let an initial game position $\{t_*, x_*\}$ be specified. We consider the following two problems /6/.

Approach problem. For any number $\varepsilon > 0$ find the first player's positional piecewise-programmed strategy φ_ε such that for all paths $(\varphi_\varepsilon = (\Delta(\varepsilon), \varphi_{\Delta(\varepsilon)}))$

$$x(t) = \varkappa(t, t_*, x_*, \varphi_\varepsilon, \psi^{\Delta(\varepsilon)}), \; \psi^{\Delta(\varepsilon)} \in D_2^{\Delta(\varepsilon)}[t_*]$$

the relations

$$\{\tau, x(\tau)\} \in M^\varepsilon, \quad \{t, x(t)\} \in N^\varepsilon, \quad t_* \leqslant t < \tau = \tau[x(\cdot)] \leqslant T \tag{6.1}$$

are fulfilled.

Evasion problem. Find a number $\varepsilon > 0$ and a second player's positional piecewise-programmed strategy ψ_ε such that contact (6.1) is excluded for all paths $(\psi_\varepsilon = (\Delta(\varepsilon), \psi_{\Delta(\varepsilon)}))$

$$x(t) = \varkappa(t, t_*, x_*, \varphi^{\Delta(\varepsilon)}, \psi_\varepsilon), \quad \varphi^{\Delta(\varepsilon)} \in D_1^{\Delta(\varepsilon)}[t_*]$$

The following theorem on the alternative /1,2,6,10/ is valid.

Theorem 4. If condition (5.2) is fulfilled, then either the Approach problem or the Evasion problem is solvable for any position $\{t_*, x_*\}$.

The first player, in the Approach problem, and the second player, in the Evasion problem, employ upper Δ-strategies. They may even use past realizations of the controls of both players. This is due to the fact that a player-ally cannot impose any restrictions on the information available to the opponent /1/. Theorems 1— 3 show that Theorem 4 on the alternative remains valid if the opponent is allowed to use recursive /8/, piecewise-synthesizing or global strategies /9/.

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